II-D (解答例)

問 1 (1)

$$
\begin{aligned}
I_1(\theta) &= \frac{E}{a\lambda(\pi-\theta)}[\mathrm{A}] \\
I_2(\theta) &= \frac{E}{a\lambda(\pi+\theta)}[\mathrm{A}]
\end{aligned}
$$

(2)

$$
R_c(\theta) = \frac{E}{I_1+I_2} = \frac{a\lambda}{2\pi}(\pi^2-\theta^2)[\Omega]
$$

問 2 (1) 全体の抵抗 R は

$$
R(\theta) = a\lambda + R_c(\theta) = \frac{a\lambda}{2\pi}(2\pi+\pi^2-\theta^2) = \frac{a\lambda}{2\pi}(\alpha^2-\theta^2)[\Omega]
$$

であるから，電流 I は

$$
I(\theta) = \frac{E}{R} = \frac{2\pi E}{a\lambda(\alpha^2-\theta^2)}[\Omega]
$$

(2)

$$
P_c(\theta) = R_c(\theta)I(\theta)^2 = \frac{2\pi E^2}{a\lambda}\frac{\left(\pi^2-\theta^2\right)}{\left(\alpha^2-\theta^2\right)^2}[\mathrm{W}]
$$

(3)

$$
\begin{aligned}
\frac{dP_c(\theta)}{d\theta} &= \frac{2\pi E^2}{a\lambda}\cdot\frac{-2\theta\left(\alpha^2-\theta^2\right)^2+4\theta(\pi^2-\theta^2)(\alpha^2-\theta^2)}{\left(\alpha^2-\theta^2\right)^4} \\
&= \frac{2\pi E^2}{a\lambda}\cdot\frac{2\theta(\alpha^2-\theta^2)(2(\pi^2-\theta^2)-(\alpha^2-\theta^2))}{\left(\alpha^2-\theta^2\right)^4} \\
&= \frac{4\pi E^2}{a\lambda}\cdot\frac{\theta(\beta^2-\theta^2)}{\left(\alpha^2-\theta^2\right)^3}
\end{aligned}
$$

したがって，$P(\theta)$ が極値を取りうるのは，$\theta = 0, \pm\beta$。これを元に増減表を作ると，

θ	$-\pi\cdots$	$-\beta$		0		$+\beta$	$\cdots+\pi$
$\frac{dP_c}{d\theta}$	$+$	0	$-$	0	$+$	0	$-$
$P_c(\theta)$	$\nearrow$		$\searrow$		$\nearrow$		$\searrow$

よって，P_c が最大となるのは $\theta_0 = \pm\beta[\mathrm{rad}]$ の時で，その時の $P_c^{\max}$ は

$$
P_c^{\max} = \frac{2\pi E^2}{a\lambda}\cdot\frac{\pi^2-\beta^2}{\left(\alpha^2-\beta^2\right)^2} = \frac{E^2}{4a\lambda}[\mathrm{W}]
$$

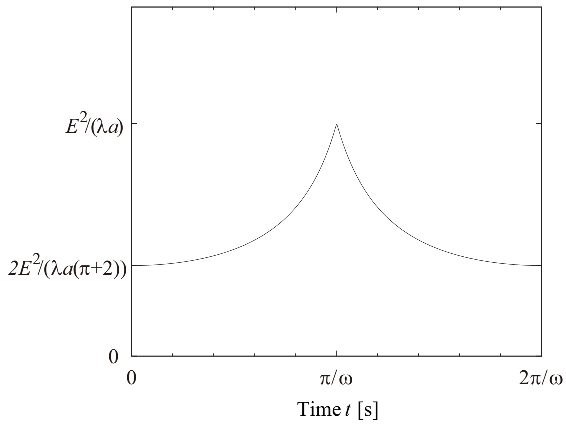

図 1: $P(t)$ の変化

問 3 (1)

$$P(t) = EI(t) = \frac{2\pi E^2}{a\lambda} \cdot \frac{1}{\alpha^2 - (\omega t)^2} [\mathrm{W}]$$

(2) 図 1 の通り。

(3) $P(t)$ の対称性を利用して,

$$\begin{aligned} W &= \int_{-\pi/\omega}^{+\pi/\omega} P(t)dt = 2\int_0^{\pi/\omega} P(t)dt \\ &= \frac{4\pi E^2}{a\lambda} \int_0^{\pi/\omega} \frac{dt}{\alpha^2 - (\omega t)^2} \end{aligned}$$

ここで、$x = \omega t$ とおけば,

$$\begin{aligned} W &= \frac{4\pi E^2}{a\lambda\omega} \int_0^{\pi} \frac{dx}{\alpha^2 - x^2} \\ &= \frac{2\pi E^2}{a\lambda\omega\alpha} \int_0^{\pi} \left(\frac{1}{\alpha + x} + \frac{1}{\alpha - x} \right) dx \\ &= \frac{2\pi E^2}{a\lambda\omega\alpha} \left[\log(\alpha + x) - \log(\alpha - x) \right]_0^{\pi} \\ &= \frac{2\pi E^2}{a\lambda\omega\alpha} \log\left(\frac{\alpha + \pi}{\alpha - \pi} \right) [\mathrm{J}] \end{aligned}$$

(4) R_e が $t = 0 \sim 2\pi/\omega[\mathrm{s}]$ の間に消費する電力量は

$$\frac{E^2}{R_e} \cdot \frac{2\pi}{\omega} [\mathrm{J}]$$

であり, これが前問の W と等しいとすれば,

$$R_e = \frac{a\lambda\alpha}{\log\left(\frac{\alpha+\pi}{\alpha-\pi}\right)} [\Omega]$$